Hints and References to Prepare for Honors Algebra 2/Trigonometry

Correlation of Hon Alg 2/Trig Summer Packet with Algebra 1 Objectives:

A. Simplifying Polynomial Expressions

Objectives: The student will be able to:
- Use the commutative, associative, identity, and distributive properties.
- Apply the appropriate arithmetic operations needed to simplify an algebraic expression.
- Simplify polynomials using addition and subtraction.
- Multiply a monomial and polynomial.

B. Solving Equations

Objectives: The student will be able to:
- Solve multi-step equations.
- Solve a literal equation for a specific variable, and use formulas to solve problems.

C. Rules of Exponents

Objectives: The student will be able to:
- Simply expressions using the laws of exponents.
- Evaluate powers that have zero or negative exponents.

D. Binomial Multiplication

Objectives: The student will be able to:
- Multiply two binomials.

E. Factoring

Objectives: The student will be able to:
- Identify the greatest common factor of the terms of a polynomial expression.
- Express a polynomial as a product of a monomial and a polynomial.
- Find all factors of the quadratic expression $ax^2 + bx + c$ by factoring and graphing.

F. Radicals

Objectives: The student will be able to:
- Add, subtract, multiply, divide and simplify square roots.
- Find the real solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ using factoring and the quadratic formula.

G. Graphing Lines

Objectives: The student will be able to:
- Identify and calculate the slope of a line.
- Graph linear equations using a variety of methods.
- Determine an equation of a line.
- Rewrite an equation in standard form to slope intercept form and vice versa.

A. Simplifying Polynomial Expressions

I. Combining Like Terms

- You can add or subtract terms that are considered "alike", or terms that have the same variable with the same exponent

EX 1: $5x - 7y + 10x + 3y$

$\underline{5x}\ \underline{\underline{-\ 7y}} + \underline{10x} + \underline{\underline{3y}}$

$15x - 4y$

EX 2: $-8h^2 + 10h^3 - 12h^2 - 15h^3$

$\underline{\underline{-8h^2}} + \underline{10h^3}\ \underline{\underline{-\ 12h^2}}\ \underline{-\ 15h^3}$

$-20h^2 - 5h^3$

II. Applying the Distributive Property

- Every term inside the parentheses is multiplied by the term outside of the parentheses.

$Ex\ 1{:}\ 3(9x-4)$

$3\cdot 9x - 3\cdot 4$

$27x - 12$

$Ex\ 2{:}\ 4x^2(5x^3+6x)$

$4x^2\cdot 5x^3 + 4x^2\cdot 6x$

$20x^5 + 24x^3$

III. Combining Like Terms AND the Distributive Property (Problems with a Mix!)

- Sometimes problems will require you to distribute AND combine like terms!!

$Ex\ 1{:}\ 3(4x-2)+13x$

$3\cdot 4x - 3\cdot 2 + 13x$

$12x - 6 + 13x$

$25x - 6$

$Ex\ 2{:}\ 3(12x-5) - 9(-7+10x)$

$3\cdot 12x - 3\cdot 5 - 9(-7) - 9(10x)$

$36x - 15 + 63 - 90x$

$-54x + 48$

B. Solving Equations

I. Solving Two-Step Equations

A couple of hints:

1. To solve an equation, UNDO the order of operations.
2. Save the operation that is directly related to the variable as the last operation you will undo.
3. REMEMBER! Addition is "undone" by subtraction, and vice versa. Multiplication is "undone" by division, and vice versa.

$$\begin{aligned} Ex\ 1:\ 4x - 2 &= 30 \\ +2 \quad & +2 \\ 4x &= 32 \\ \div 4 \quad & \div 4 \\ x &= 8 \end{aligned}$$

$$\begin{aligned} Ex\ 2:\ 87 &= -11x + 21 \\ -21 \quad & -21 \\ 66 &= -11x \\ \div -11 \quad & \div -11 \\ -6 &= x \end{aligned}$$

II. Solving Multi-step Equations With Variables on Both Sides of the Equals Sign

- When solving equations with variables on both sides of the equal sign, be sure to get all terms with variables on one side and all the terms without variables on the other side.

$$\begin{aligned} Ex\ 3:\ 8x + 4 &= 4x + 28 \\ -4 \quad & \quad -4 \\ 8x &= 4x + 24 \\ -4x \quad & -4x \\ 4x &= 24 \\ \div 4 \quad & \div 4 \\ x &= 6 \end{aligned}$$

III. Solving Equations that need to be simplified first

- In some equations, you will need to combine like terms and/or use the distributive property to simplify each side of the equation, and then begin to solve it.

$$\begin{aligned} Ex\ 4:\ 5(4x - 7) &= 8x + 45 + 2x \\ 20x - 35 &= 10x + 45 \\ -10x \quad & -10x \\ 10x - 35 &= 45 \\ +35 \quad & +35 \\ 10x &= 80 \\ \div 10 \quad & \div 10 \\ x &= 8 \end{aligned}$$

IV. Solving Literal Equations

- A literal equation is an equation that contains more than one variable.
- You can solve a literal equation for one of the variables by getting that variable by itself (isolating the specified variable).

$Ex\ 1:\ 3xy = 18,\ solve\ \ for\ \ x$

$$\frac{3xy}{3y} = \frac{18}{3y}$$

$$x = \frac{6}{y}$$

$Ex\ 2:\ 5a - 10b = 20$

$$+10b \quad = \quad +10b$$

$$5a = 20 + 10b$$

$$\frac{5a}{5} = \frac{20}{5} + \frac{10b}{5}$$

$$a = 4 + 2b$$

C. Exponent Rules

Multiplication: Recall $(x^m)(x^n) = x^{(m+n)}$

Ex: $(3x^4y^2)(4xy^5) = (3\cdot 4)(x^4\cdot x^1)(y^2\cdot y^5) = 12x^5y^6$

Division: Recall $\dfrac{x^m}{x^n} = x^{(m-n)}$

Ex: $\dfrac{42m^5j^2}{-3m^3j} = \left(\dfrac{42}{-3}\right)\left(\dfrac{m^5}{m^3}\right)\left(\dfrac{j^2}{j^1}\right) = -14m^2j$

Powers: Recall $(x^m)^n = x^{(m\cdot n)}$

Ex: $(-2a^3bc^4)^3 = (-2)^3(a^3)^3(b^1)^3(c^4)^3 = -8a^9b^3c^{12}$

Power of Zero: Recall $x^0 = 1,\ x \neq 0$

Ex: $5x^0y^4 = (5)(1)(y^4) = 5y^4$

D. Binomial Multiplication

I. Reviewing the Distributive Property

The distributive property is used when you want to multiply a single term by an expression.

Ex 1: $8(5x^2 - 9x)$

$$8 \cdot 5x^2 + 8 \cdot (-9x)$$

$$40x^2 - 72x$$

II. Multiplying Binomials – the FOIL method

When multiplying two binomials (an expression with <u>two</u> terms), we use the "FOIL" method. The "FOIL" method uses the distributive property <u>twice</u>!

FOIL is the order in which you will multiply your terms.

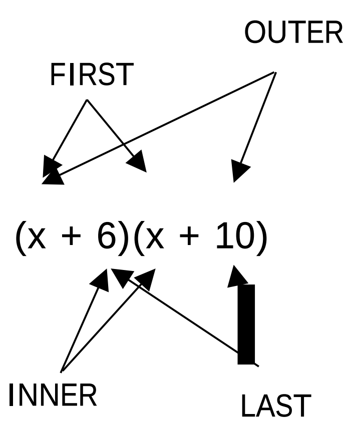
First

Outer

Inner

Last

Ex 1: (x + 6)(x + 10)



First	$x \cdot x$ ------> x^2
Outer	$x \cdot 10$ -----> $10x$
Inner	$6 \cdot x$ ------> $6x$
Last	$6 \cdot 10$ -----> 60

$$x^2 + 10x + 6x + 60$$

$$x^2 + 16x + 60$$

(After combining like terms)

NOTE: Special Case

Recall: $4^2 = 4 \cdot 4$

$x^2 = x \cdot x$

Ex. $(x + 5)^2$

$(x + 5)^2 = (x + 5)(x+5)$

Now you can use the "FOIL" method to get a simplified expression.

E. Factoring

I. Determining the greatest common factor (GCF).

- Always determine whether there is a greatest common factor (GCF) first.

Ex. 1 $3x^4 - 33x^3 + 90x^2$

- In this example the GCF is $3x^2$.
- So when we factor, we have $3x^2(x^2 - 11x + 30)$.

- Now we need to look at the polynomial remaining in the parentheses. Can this trinomial be factored into two binomials? In order to determine this make a list of all of the factors of 30.

30

1	30
2	15
3	10
5	6

Since -5 + -6 = -11 and (-5)(-6) = 30 we should choose -5 and -6 in order to factor the expression.

- The expression factors into $3x^2(x-5)(x-6)$

Note: Not all expression will have a GCF. If a trinomial expression does not have a GCF, proceed by trying to factor the trinomial into two binomials.

II. Applying the difference of squares: $a^2 - b^2 = (a-b)(a+b)$

Ex. 2

$4x^3 - 100x$

$4x(x^2 - 25)$

$4x(x-5)(x+5)$

Since x^2 and 25 are perfect squares separated by a subtraction sign, you can apply the difference of two squares formula.

F. Radicals

To simplify a radical, we need to find the greatest perfect square factor of the number under the radical sign (the radicand) and then take the square root of that number.

Ex 1: $\sqrt{72}$

$\sqrt{36} \cdot \sqrt{2}$

$6\sqrt{2}$

Ex 2: $4\sqrt{90}$

$4 \cdot \sqrt{9} \cdot \sqrt{10}$

$4 \cdot 3 \cdot \sqrt{10}$

$12\sqrt{10}$

Ex 3: $\sqrt{48}$

$\sqrt{16}\sqrt{3}$

$4\sqrt{3}$

OR

Ex 3: $\sqrt{48}$

$\sqrt{4}\sqrt{12}$

$2\sqrt{12}$

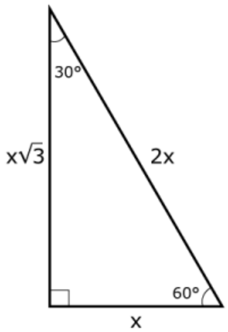
This is not simplified completely because 12 is divisible by 4 (another perfect square)

$2\sqrt{4}\sqrt{3}$

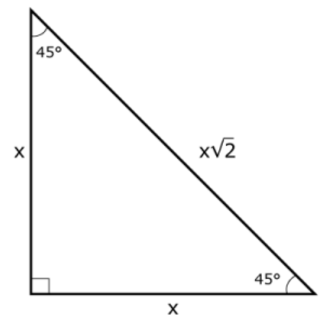
$2 \cdot 2 \cdot \sqrt{3}$

G. Special Right Triangles & Trig

I. To find the missing side of these special right triangle use the below ratios:

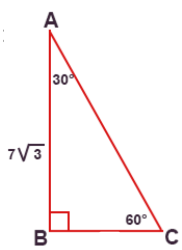


30-60-90



45-45-90

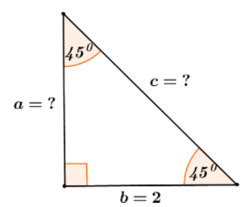
Ex 1:



Find the lengths of BC and AC.

Since the side opposite 60° is x$\sqrt{3}$ then that means x = 7, making side ***BC = 7***. Now, we know that the hypotenuse is double the side opposite 30°, so 7 x 2 = 14, making ***AC = 14***.

Ex 2:



Find the lengths of a and c.

Since we know that both legs of a 45-45-90 Δ are equal, if b = 2, then ***a = 2*** as well. The hypotenuse is the length of the leg multiplied by $\sqrt{2}$, so ***c = 2***$\sqrt{2}$.

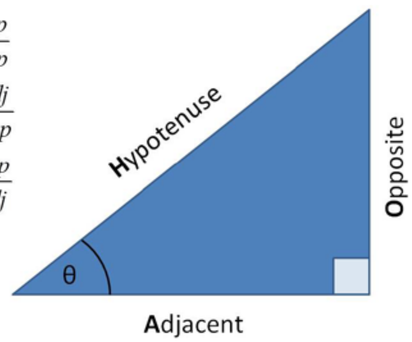
II. To solve for a missing side or angle of any right triangle use:

SOHCAHTOA

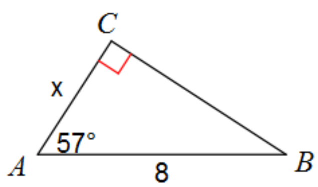
$$\sin(\theta) = \frac{Opp}{Hyp}$$

$$\cos(\theta) = \frac{Adj}{Hyp}$$

$$\tan(\theta) = \frac{Opp}{Adj}$$



Ex 1: Solve for a side



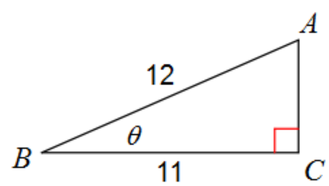
Because we are given angle A, that makes x the adjacent side and 8 is the hypotenuse. The trig function we need is cos given adj and hyp. So,

$\cos(57) = \frac{x}{8}$ Multiply both sides by 8

$8 * \cos(57) = x$ Be sure you are in DEG mode!

x = 4.4

Ex 2: Solve for an angle



When missing an angle, we set up the formula the same way. We want to find angle B. In reference to <B, 11 is the adjacent side and 12 is the hypotenuse. So,

$\cos(\theta) = \frac{11}{12}$ Use $\cos^{-1}$ on calc

$\cancel{\cos^{-1}(\cos}(\theta)) = \cos^{-1}(\frac{11}{12})$ DEG mode!

θ = 23.6°